Microwave Imaging Method Using a Simulated Annealing Approach

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***Abstract*—A new method of microwave imaging in the space domain is presented, which is based on a simulated annealing approach. Markov random fields are used to model the distributions of dielectric features, and a stochastic relaxation algorithm is developed to solve the imaging problem. Computationally heavy inversions of large matrices are avoided. Numerical results confirming the capabilities of the proposed method are reported: they seem better than those obtained by other imaging techniques in space domain.**

I. Introduction

The methods of microwave imaging in the space domain (SD-MI) seem to be very promising [1], but they are ill-conditioned and expensive in terms of computations. We propose a new method if SD-MI, based on a simulated annealing approach [2] coupled to a stochastic relaxation algorithm [3]. This bypasses the need for inverting large matrices, and allows one to obtain the solution by using an iterative procedure that gives the solution in few steps.

II. Mathematical Modeling

Let us consider a two-dimensional scattering model. For TM incidence and within the Born approximation, the following equation [4] holds for the scattered field E_{scatt}:

$$E_{\text{scatt}}(x, y) = \frac{k_0^2}{j4} \int \tau(x', y') E_{\text{inc}}(x', y') H_0^{(2)}\left(k_0 \sqrt{(x - x')^2 + (y - y')^2}\right) dx'\, dy' + \eta, \quad (1)$$

where η is a noise term, standing for both measurement and computational noise. This equation is discretized into a regular grid of N cells (pixels), with M measurement points [4]. This leads to the following formulation:

$$E_{\text{scatt}}^m = \sum_{n=1}^{N} \tau_n E_{\text{inc}}^n H_0^{m,n} + \eta_m, \qquad m = 1, \cdots, M. \quad (2)$$

We aim to reconstruct the object function τ by maximizing the *a posteriori* probability:

$$P(\tau / E_{\text{scatt}}) = \frac{P(E_{\text{scatt}} / \tau) P(\tau)}{P(E_{\text{scatt}})}. \quad (3)$$

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If η is Gaussian, with zero-mean and variance σ^2, we have:

$$P(E_{\text{scatt}} / \tau) = (2\pi\sigma^2)^{-M/2} \cdot e^{-\frac{1}{2\sigma^2} \sum_{m=1}^{M} | E_{\text{scatt}}^m - \sum_{n=1}^{N} \tau_n E_{\text{inc}}^n H_0^{m,n} |^2}. \quad (4)$$

It is realistic to model τ by a Markov random field (MRF) [3], thus its probability becomes a Gibbs one and can be written as:

$$P(\tau) = \frac{e^{-\sum_n \sum_j | \tau_n - \tau_j |^2}}{\sum_{\text{all possible configurations}}} = \frac{e^{-\sum_n \sum_j | \tau_n - \tau_j |^2}}{Z}, \qquad n = 1, \cdots, N, \quad (5)$$

where j is included in a first-order neighbor system [3] of n for all n.

Maximizing (3) is equivalent to minimizing the function:

$$U(\tau) = \beta \sum_{n=1}^{N} \sum_{j} | \tau_n - \tau_j |^2 + \sum_{m=1}^{M} \left| E_{\text{scatt}}^m - \sum_{n=1}^{N} \tau_n E_{\text{inc}}^n H_0^{m,n} \right|^2 = \beta \sum_{n=1}^{N} \sum_{j} | \tau_n - \tau_j |^2 + V_{\text{scatt}}(\tau) \quad (6)$$

where β includes all constants and can be regarded as a regularization parameter for the system.

To minimize this expression, a simulated annealing approach [2] coupled to a stochastic relaxation algorithm [3] has been used. Stochastic relaxation can be combined with simulated annealing by using a suitable control parameter T (i.e., the "temperature of the system"), which is slowly decreased during the sampling process. The proposed method requires the following steps.

Choose an initial configuration for τ and compute U.
  Let $T = T_{\text{start}}$
  ① $n = 1$.
  ② Randomly choose a new τ_n from among the discrete set of possible values and compute U_{new}
  $\Delta U = U_{\text{new}} - U$
  IF $\Delta U < 0$ THEN
    always accept this new τ_n value
    $U = U_{\text{new}}$

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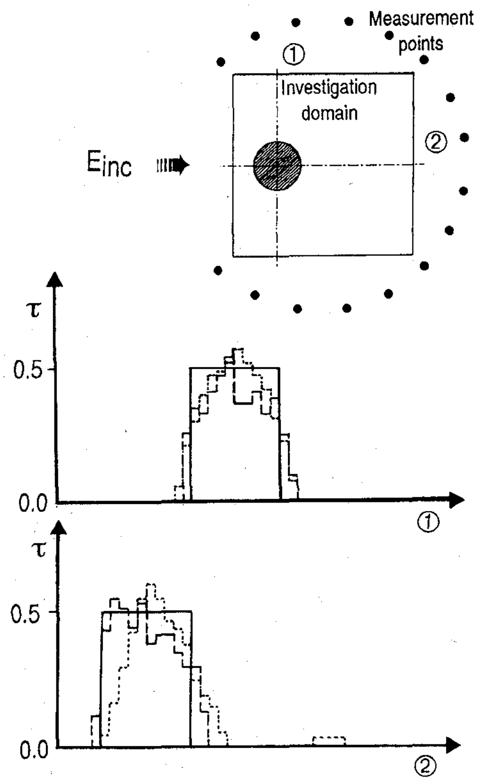

Fig. 1. First simulation case (circular cylinder). (a) Geometric arrangement. (b) Reconstruction behavior along the vertical Section 1. Dotted line: 1 view; dashed line: 4 views. (c) Reconstruction behavior along the horizontal Section 2. Continuous line: 1 view; dashed line: 4 views.

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accept this new τ_n value with probability $p = e^{-\Delta U/T}$

$U = U_{\text{new}}$

or

reject this new τ_n value with probability $q = 1 - p$

$n = n + 1$

IF $n \leq N$ GO TO ②

IF the convergence criterion is satisfied THEN

STOP

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decrease T by using a suitable scheduling (not discussed here)

GO TO ① (start a new iteration step).

As can be noticed, the algorithm performs a purely random search for infinite T, whereas it tends to become deterministic as T approaches 0.

III. Numerical Simulations and Results

In all the simulations we used an illuminating plane wave at a frequency of 10 GHz, and a square investigation domain of $\lambda_0 \times \lambda_0$, subdivided into 40 × 40 square pixels. 16 measurement points were used. The background medium was vacuum.

In the first set of simulations we considered a circular cylinder with a complex dielectric permittivity $\epsilon_r = 1.5 - j0$, that is $\tau = 0.5 - j0$, located inside the investigation domain as shown in Fig. 1. Fig. 1 give the behaviors of the reconstructions of the real part of ϵ_r along a vertical and a horizontal section of the investigation domain; both sections pass through the center of the scattering cylinder. The graphs show the behaviors of 1- and 4-view reconstructions, together with the ideal behavior. As can be noticed, reconstruction results are satisfactory for both 1 and 4 views, even though it appears evident that a single view causes a larger error on the localization of the scatterer. Fig. 2 gives the behaviors of the whole energy function $U(\tau)$ and of the part related only to the error on the scattered field $V_{\text{scatt}}(\tau)$ versus

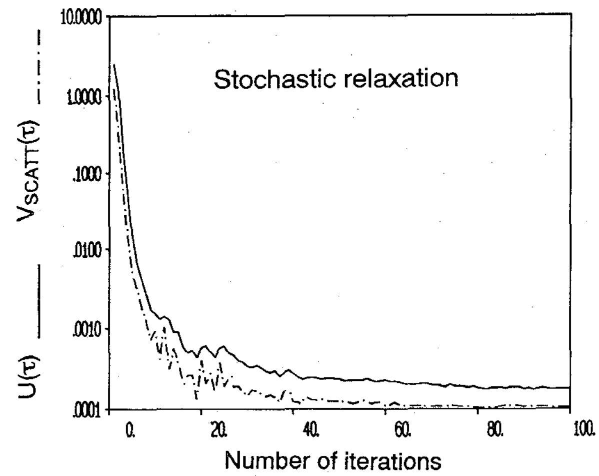

Fig. 2. First simulation geometry (circular cylinder); 1 view. Behaviors of the whole function $U(\tau)$ (continuous line) and of the term $V_{\text{scatt}}(\tau)$ related only to the error on the scattered field (dashed line) versus the number of iterations. The variations in Re$\{\epsilon_r\}$ were *a priori* assumed to range between 1 and 4.

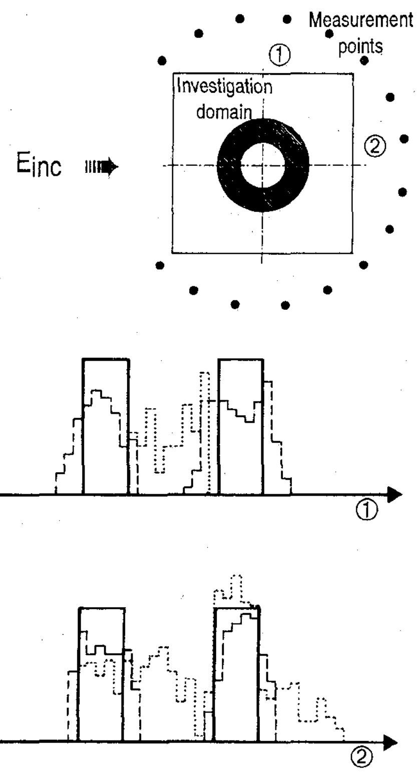

Fig. 3. Second simulation case (circle ring). (a) Geometric arrangement. (b) Reconstruction behavior along the vertical Section 1. Dotted line: 1 view; dashed line: 4 views. (c) Reconstruction behavior along the horizontal Section 2. Continuous line: 1 view; dashed line: 4 views.

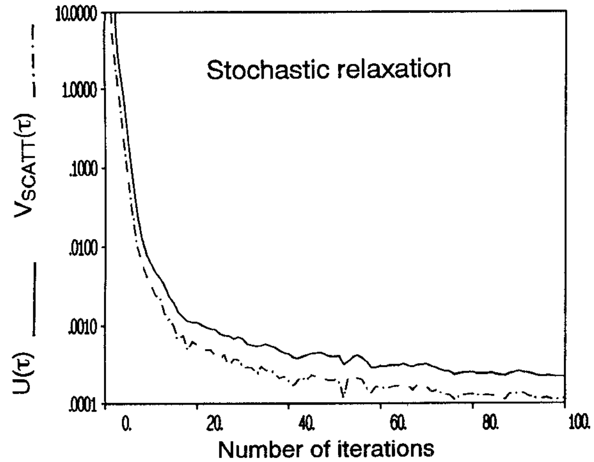

Fig. 4. First simulation geometry (circular cylinder); 1 view. Behaviors of the whole function $U(\tau)$ (continuous line) and of the term $V_{\text{scatt}}(\tau)$ related only to the error on the scattered field (dashed line) versus the number of iterations. In this case, the variations in Re $\{e_r\}$ were *a priori* assumed to range between 1 and 11.

the increasing number of process iterations. Such graphs show damped oscillatory behaviors, which are specific for the algorithm used to reach convergence. Results have been obtained by 1000 iterations; however, it is worth noting that, after about 40 iterations (requiring few seconds of processing), reconstructions results are already acceptable.

In the second set of simulations, a circle ring was considered, with the same dielectric permittivity (i.e., $\epsilon_r = 1.5 - j0$). Results are shown in Fig. 3. In this case it can be seen that the use of a single view is not enough to obtain a good results; and that there is a sharp difference between the monoview reconstruction and the 4-view one, the latter being far more accurate. The obtained results are better than those yielded by other SD-MI algorithms [1], [4], [5], especially as regards the reconstruction of the circle ring. Another interesting aspect of the proposed method versus other algorithms, lies in the possibility of easily including *a priori* knowledge in the reconstruction algorithm. This allows one to avoid reconstructed values of ϵ_r outside a reasonable range. In particular, for these simulations, we assumed, as *a priori* knowledge, that the variations in Re $\{\epsilon_r\}$ ranged between 1 and 4. This range is sufficient in many real situations; however, the first monoview simulation was repeated, assuming a range from 1 to 11, that is a very high contrast. In this case, too, results are good, even though convergence is slightly slower, as can be deduced from Fig. 4.

IV. Conclusion

A new method of SD-MI has been presented, which is based on a simulated annealing algorithm. The use of an MRF model allows one to take into account *a priori* information about the nature of the scatterers involved. Numerical results have proved the efficiency of the method in imaging simple objects, within the Born approximation. However, the proposed scheme can be further improved, and a version using the exact inverse-scattering equation for strong scatterers is currently under development.

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